One-dimensional controllable photonic crystal

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The nonlinear one-dimensional model of controllable photonic crystals is considered. A photonic crystal is formed in a dielectric film on which the electromagnetic pump and signal waves fall. It is shown that the shape of the superlattice is controlled by the variation of the external electrostatic field imposed on the film. Thus it leads to changing boundaries of the allowed and forbidden zones of the signal wave spectrum. © 2007 Optical Society of America

1. INTRODUCTION

One-, two- or three-dimensional nonlinear macroscopic structures arise in the media with a continuous inflow of energy from an exterior source, with the reflux and dissipation of the energy [1]. Such macroscopic structures are the solitary and periodic stationary spatial waves, the optical vortices, the spiral waves, etc., that appear in the dielectric medium by passing the electromagnetic field. The shape-incipient macroscopic object depends on properties of the dielectric medium and parameters of the external electromagnetic field.

Photonic crystals are the dielectric periodic structures with forbidden regions (bandgaps) in an optical spectrum, and they are recently actively explored [1,2]. The photonic crystals were made so that even one of the sizes of their periodic structure was about the length of a light wave. Thus they can function in both the linear and the nonlinear regime; the intensity of the electromagnetic field and the structure period determine the spectrum profile [3].

At the interaction of the electromagnetic field with the medium, the quasi-particles named polaritons are generated in the nonlinear crystals and in the nonlinear-amorphous dielectric media. In case the dielectric medium has the boundaries, the polaritons in one- and two-dimensional models form the Bose–Einstein quasi-condensate at a cross section of the resonator longitudinal axis [4]. The polaritons are at the ground state in the quasi-condensate when they propagate strictly along the axis of the dielectric resonator, and they have the zero-cross impulse components. The excitations of the quasi-condensate form the stationary spatial waves with the zero of amplitude on the resonator transverse axis [5]. In the areas around the wave maxima the refractive index of the medium increases and around the wave minima practically does not vary [6]. Thus in the dielectric medium the superlattice is formed. The shape of the superlattice can be controlled by changing parameters of the electromagnetic field.

If the signal electromagnetic wave falls on the dielectric medium with the superlattice, then it will be scattered on the inhomogeneities of the medium, i.e. on the periodic potential. For the signal wave there will be the areas of the allowed and the forbidden frequencies with the changeable boundaries by the varying of the external electrostatic field. Thus, in the dielectric medium the controllable photonic crystal is formed.

2. BOSON HAMILTONIAN

We shall consider the quantum processes of interaction between the electromagnetic wave and the nonlinear amorphous dielectric medium in the volumetric unclosed resonator. Let us write the equation for the classical electric field $E$ in medium $V^2E - \nabla (\nabla E) - 1/c^2 \nabla^2 E = 0$, where $\varepsilon=\varepsilon_1+4\pi\chi_3\epsilon_0 E$ is the nonlinear permittivity, $\varepsilon_1$ is the linear permittivity, and $\chi_3$ is the cubic susceptibility of medium. We shall use the electromagnetic field potentials in Hamilton gauge $\Phi=0$, $E=-c^{-1}\partial A/\partial t$, and we obtain the equation for the vector potential:

$$V^2A - \nabla (\nabla A) - \frac{1}{c^2} \left( \frac{\partial}{\partial t} \frac{\partial}{\partial t} \right) A = 0. \quad (1)$$

The last term in Eq. (1) we shall rewrite as

$$\frac{1}{c^2} \left( \frac{\partial}{\partial t} \frac{\partial}{\partial t} \right) A = \frac{1}{c^2} \left( \frac{\partial}{\partial t} \frac{\partial A}{\partial t} \right) + \frac{4\pi\chi_3}{c^4} \left( \frac{\partial^2 A}{\partial t^2} \frac{\partial A}{\partial t} + 2 \frac{\partial A}{\partial t} \frac{\partial^2 A}{\partial t^2} \right),$$

where the linear dispersion $\varepsilon_1=\varepsilon_1(\omega)$ is taken into account, and the nonlinear response of medium $\chi_3=const.$ is small. For the quasi-monochromatic field $\Delta\omega \ll \omega$ we shall write the linear response like the Brillouin operator [7]

$$\left( \frac{\partial}{\partial t} - \frac{\varepsilon_1}{\varepsilon_0} \frac{\partial}{\partial t} \right) \hat{A}(\omega + \Delta\omega).$$

Then we expand the operator $\hat{f}$ that acts at the quasi-monochromatic field $A(t,\mathbf{r})=\hat{A}(t,\mathbf{r})e^{i\omega t}$ in the Taylor series:

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\[ \hat{\mathbf{A}}(\omega + \Delta \omega) = \{ \hat{\mathbf{A}}(\omega) + \hat{\mathbf{A}}(\omega + \Delta \omega) \} = \{ \hat{\mathbf{A}}(\omega) \} + \hat{\mathbf{A}}(\omega + \Delta \omega) \exp(-i \Delta \omega t), \]

where \( \hat{\mathbf{A}}(\omega) = -\alpha^2 e_1 \). If we suppose that the field is changing slowly enough for \( T_0 \ll |\alpha|/\Delta \omega | \ll 1 \) for the period \( T_0 \), then the first approximation gives the system of equations for slowly varying amplitudes of the potential \( \hat{\mathbf{A}}(\omega) = (\psi_1, \psi_2, \psi_3) \):

\[
\frac{\partial \psi_j}{\partial t} = -\alpha_0 \nabla^2 \psi_j + \alpha_0 \psi_j \left( \sum_{j'} \alpha_{j} \psi_{j'} - \alpha_2 \sum_{j'} \psi_j \psi_{j'} \right) \psi_j - \alpha_2 \left( \sum_{j'} \psi_j \psi_{j'} \right) \psi_j,
\]

where \( \alpha_0 = c_0 \{ \frac{\partial |d_0|^2}{\partial \omega e_1} \} / (\partial \omega e_1) \}, \alpha_1 = e_1 \omega^2 \{ \frac{\partial |d_0|^2}{\partial \omega e_1} \} / (\partial \omega e_1) \}, \alpha_2 = 4 \pi \nu \omega^2 \{ \frac{\partial |d_0|^2}{\partial \omega e_1} \} / (\partial \omega e_1) \}, \) and \( j = 1, 2, 3 \). The Lagrangian of the system is

\[
\tilde{L} = \frac{\omega}{c^2} \sum_j \left[ i \psi_j \psi_j^* / \partial t + \alpha_0 \left( \nabla \psi_j - \mathbf{1} \sum_{j'} \partial \psi_{j'} / \partial t \psi_j \right) \psi_j^* - \frac{\alpha_1}{2} \frac{\partial \psi_j}{\partial t} \psi_j \psi_j^* \right] - \frac{\alpha_2}{16} (\psi_j \psi_j^* + \psi_j^* \psi_j)^2 + \text{c.c.}.
\]

We shall write the Hamiltonian of the system

\[
H = \int dV \sum_{j, \mu=0,1,2,3} \left( \psi_j^* \partial \tilde{L} / \partial \psi_j^* \right) - \tilde{L}
\]

by the field operators for the amorphous medium in the resonator with the volume \( V_0 \):

\[
\psi_j(t, \mathbf{r}) = \frac{\sqrt{\hbar}}{2\omega V_0} \sum_k \alpha_{jk} e^{-(i\omega+\Delta \omega)t} \exp(ik\mathbf{r}),
\]

\[
\psi_j^*(t, \mathbf{r}) = \frac{\sqrt{\hbar}}{2\omega V_0} \sum_k \alpha_{jk}^* e^{-(i\omega+\Delta \omega)t} \exp(-ik\mathbf{r}).
\]

Then we obtain the Hamiltonian in the boson operator with the relations \( \{ \alpha_{jk}, \alpha_{jk}^* \} = 0 \):

\[
\hat{H} = \sum_{j,k} \left\{ \tilde{\omega}(k + \tilde{\alpha}) \sum_{j'} \left( \alpha_{k,j}^* \alpha_{j,k} + \frac{1}{2} \right) \right\} \left( \alpha_{k,j}^* \alpha_{j,k} + \frac{1}{2} \right) - \frac{\alpha_0}{2} \sum_{j,k} \sum_{j'} \left( \alpha_{k,j} \alpha_{j,k}^* + \alpha_{j,k} \alpha_{k,j}^* \right),
\]

where \( \tilde{\omega}(k) = \omega + \Delta \omega + \alpha_1 + \alpha_2 k^2, \tilde{\alpha} = \hbar^2 \alpha_2 / 4 \omega V_0 \). The boson Hamiltonian in form of Eq. (3) is well known in the quantum field theory [9]. In this case the Hamiltonian (3) describes the bound states of photons in the amorphous dielectric medium, i.e., the polaritons.

3. POLARITON QUASI-CONDENSATE

Let us consider Chiao’s model “electromagnetic field—dielectric medium in the resonator” [4]. The dielectric medium is placed in the unclosed Fabry–Perot resonator with the flat mirrors. The electromagnetic wave falls on the input mirror of the resonator and then leaves through the output mirror. As the resonator has such a high quality factor the polaritons can stay in the resonator long enough to establish the statistical equilibrium. The quadratic of the wave number of \( \ell \)th resonator mode is equal to

\[ k_\ell^2 = k_{\ell i}^2 + k_{\ell j}^2, \]

where \( k_{\ell i} = \ell / \pi L \) is a propagation constant of a mode along the longitudinal axis \( 3 \rightarrow z \) of the resonator, \( L \) is the length of the resonator, and \( \ell = \pm 1, \pm 2, \ldots \). The cross-impulse components of the polariton that propagate strictly along the \( z \) axis are equal to zero \( h k_{\ell i} = h k_{\ell j} \). We can calculate the energy system changes by the introduction of the effective chemical potential \( \mu \), because the average number of quasi-particles is stable in the resonator. Chemical potential is introduced as the Lagrange factor in one- or two-dimensional models.

The change of the wave vector \( \mathbf{k} = (k_1, k_2, \ell / \pi L) \) of the polariton impulse \( \mathbf{p} = h \mathbf{k} \), is taken into account only in a cross plane.

The Hamiltonian is \( \hat{H} = \hat{H}_0 + \hat{H}_1 + \hat{H}_2 \),

\[
N = \sum_j N_{j0} + \langle 0 \sum_{j,k \neq 0} a_{jk}^* a_{jk} \rangle, \quad \mu = \partial \tilde{E}_0 / \partial N
\]

is the chemical potential, and \( \tilde{E}_0 = \langle 0 \{ \hat{H} \} 0 \rangle \) is the energy of the ground state.

Then we consider the mode with polarization along the transverse axis. In the Hamiltonian (3) we shall allocate the terms with zero cross impulse \( p_\perp = 0 \), taking into account only the pair interactions of the polaritons \( \mathbf{k}(1) + \mathbf{k}(2) = \mathbf{k}'(1) + \mathbf{k}'(2) \) with the opposite wave vectors \( \mathbf{k} \) and \( -\mathbf{k} \).

\[
\hat{H}_0 = h \left\{ \tilde{\omega}'(0) \left( a_{00}^* a_0 + \frac{1}{2} \right) + \alpha \left( a_{00}^* a_0 + \frac{1}{2} \right) \right\},
\]

\[
\hat{H}_1 = h \sum_{k \neq 0} \tilde{\omega}'(k) \left( a_{k0}^* a_k + \frac{1}{2} \right),
\]

\[
\hat{H}_2 = h \sum_{k \neq 0, k' \neq 0} \left( a_{k0}^* a_{k'} + \frac{1}{2} \right) \left( a_{k'}^* a_k + \frac{1}{2} \right),
\]

\[
\tilde{\omega}'(k) = \omega + \Delta \omega + \alpha_1 + \alpha_2 (k_1^2 + k_2^2).
\]

We shall assume that the majority of polaritons are in the ground state with zero cross impulse \( p_\perp = 0 \). In the state with zero impulse the actual bosons form the Bose–Einstein condensate, but in one- or two-dimensional systems the Bose–Einstein condensation is not possible [10]. As the polaritons are the quasi-particles, it is possible to speak about the quasi-condensation in one- or two-dimensional systems like the unclosed resonator, considering that the number of quasi-particles is very large (\( N_0 \gg 1 \)). Then the eigenvalues of the operators \( a_0 \) and \( a_0^* \) are approximately equal (\( \sqrt{N_0} \approx 1 = \sqrt{N_0} \)). We obtain the Hamiltonian by exchanging in Eq. (4) the operators with zero cross impulse by their eigenvalues [9], within the terms of the second order of smallness:
Then we obtain the spectrum of Bogolubov type weakly biers that are symmetric at \( u \):

\[
E_\tilde{\omega} = \frac{\hbar}{2\tilde{\Omega}(k)} \text{ and } E_\tilde{\omega} = \frac{\hbar}{2\tilde{\Omega}(k) + \hbar}(N_0 + 1/2).
\]

We shall have the Hamiltonian (5) in the diagonal form

\[
\hat{H} = \tilde{E}(0) + \hbar \sum_{k=0} \left[ \Omega(k)(a_k^2 + \text{a}_k^2) + 1 \right],
\]

where \( \tilde{E}(0) = \hbar \tilde{\omega}'(k |_{\perp} = 0) + \hbar(N_0 + 1/2)](N_0 + 1/2) \), \( \Omega(k) = \tilde{\omega}'(k + \hbar(N_0 + 1/2)) \).

\[ u_k v_k = -\frac{\hbar N_0}{2\tilde{\Omega}(k)} a_k^2 = \frac{1}{2} \left( \frac{2}{\tilde{\Omega}(k)} + 1 \right), \quad v_k^2 = \frac{1}{2} \left( \frac{\Omega(k)}{\tilde{\Omega}(k) - 1} \right).
\]

Then we obtain the spectrum of Bogolubov type weakly excitations above the ground state:

\[
\tilde{\Omega}(k) = \left[ \left( \tilde{\omega}' \right)^2 + 2\tilde{\omega} \tilde{\omega}' \right]^{1/2},
\]

and the energy of ground state

\[
\tilde{E}_0 = \tilde{E}(0) + \hbar \sum_{k=0} \tilde{\Omega}(k).
\]

The chemical potential is \( \mu = \partial \tilde{E}_0 / \partial N_0 = \hbar \tilde{\omega}'(k = 0) \) in the model.

4. GROSS–PITAEVSKII EQUATION FOR POLARITON QUASI-CONDENSATE

We shall write the nonlinear Schrödinger equation for the wave function of the polariton quasi-condensate:

\[
\left( \frac{\hbar^2}{2m} \nabla^2 + \mu \right) \psi(t, r) + \left[ \int \psi(t, r') \psi(t, r') U(r - r') dV' \right] \psi(t, r).
\]

Let us assume that the density of the quasi-condensate slowly varies at the wave length of the electromagnetic beam. Then in Eq. (6) it is possible to put \( m_{ef} = \hbar \tilde{\Omega}(k)^2 \), \( \psi(t, r) \psi(t, r') U(r - r') dV' = \tilde{\Omega}(k) \tilde{\Omega}(k) \psi(t, r) \) and to receive the Gross–Pitaevski equation (9):

\[
\frac{\hbar^2}{2m_{ef}} \psi(t, r) + \hbar \tilde{\omega}' \psi(t, r) \psi(t, r) + \hbar \tilde{\omega}' \psi(t, r) \psi(t, r) \psi(t, r).
\]

We shall present a wave function of excitations as \( \psi(t, r) = \psi(r) \exp(-i\tilde{\Omega}t) \), and we obtain the equation for the function \( \psi(r) \) that depends only on the coordinates by the substituting wave function in Eq. (7):

\[
\nabla^2 \tilde{\psi} + g_1 \tilde{\psi} - g_2 \tilde{\psi}^2 \tilde{\psi} = 0,
\]

where the coefficients are \( g_1 = 2\tilde{\Omega}(\hbar\tilde{\Omega} + \mu)/\hbar^2 c^2 \) and \( g_2 = 2\hbar \alpha / c^2 \).

5. DIELECTRIC FILM AS FABRY–PEROT RESONATOR

We shall consider a process of quantum spatial-wave formation at the polariton quasi-condensate in a thin amorphous dielectric film. The dielectric film with light reflecting cross edges represents the one-dimensional Fabry–Perot resonator (Fig. 1). The electromagnetic beam falls at the input “mirror” and leaves through the output “mirror” of the resonator. The electrostatic field \( E_0 \) is positioned along the transverse axis of the resonator in the film plane. The electrostatic field \( E_0 \) allows us to control the permissivity of the medium. In such a resonator the polaritons are generated as a result of photon interaction with the dielectric medium. As the resonator has the high quality factor, the polaritons should stay in the resonator long enough to establish the statistical equilibrium.

For one-dimensional model \( k = 1, k + 1, \ell \pi / L \) we take into account the changing of the polariton impulse \( p = \hbar k \) only along the transverse axis \( z \). The quadrature of the wave vector of \( \ell \) mode of the resonator is equal to \( k^2 = k_{et}^2 + k_{eic}^2 \), where \( k_{et} \) is the cross component of the wave vector, \( k_{eic} = \ell \pi / L \) is the propagation constant of the mode along the longitudinal axis, and \( L \) is the length of the resonator, \( \ell = \pm 1, \pm 2 \ldots \). The cross component of the polariton impulse is equal to zero \( (h k_{et} = 0) \) if the polariton is propagating along the \( z \) axis. The permissivity of the film is \( \varepsilon = 1 + 4 \pi \chi_1 \omega + \chi_2 E_0 \), where \( \chi_1 \omega \) is the linear susceptibility of medium, \( \chi_2 \) is the square-law susceptibility of medium that describes the linear electro-optical Pockels effect.

We receive the equation for the wave function \( \psi = \psi(X) \) of the 1D model from Eq. (8):

\[
\frac{d^2 \psi}{dX^2} + f - f_0 = 0,
\]

where \( \psi_0 = \sqrt{g_1 / g_2} \), \( X = x / \sqrt{\eta_0} \), and \( g_1 = g_1 - c^2 / 2L^2 \). We obtain the solution of Eq. (9) as spatial cnoidal wave \( f = b_2 s + b_1 / \sqrt{\eta_0} \), \( b_2 = b_2 s + b_1 / \sqrt{\eta_0} \) [6], where \( s(X) \in [0, K(k)] \) is the elliptic sine of Jacobi, \( k = b_2 / b_1 \) is the module of the elliptic integral of the first order \( K(k) \), \( 0 < k^2 < 1 \), \( 0 < f < b_2 < b_1 \), \( b_2 = b_2 = b_2 s + b_2 s + b_2 s + b_2 s + b_2 s = 2 \).

We shall estimate the distance between zeros \( x_0 = 1 / \sqrt{\eta_0} \) of the spatial density wave \( f^2 \) in the polariton quasi-condensate. For the laser radiation with frequency \( \omega = 10^{15} \text{ s}^{-1} \) in the medium with parameters \( \varepsilon_1 = 1.5 \), \( \chi_2 \sim 10^{-20} \text{ cm}^3 / \text{erg} \) we obtain value \( x_0 \sim 10^{-5} \text{ cm} \). The fre-
quency of radiation, dispersion properties of medium, the length of resonator \( L \), and the intensity of exterior electrostatic field \( E_0 \) determine the distance between zeros of the spatial density wave in the quasi-condensate. This distance can be as small as the principle of indeterminacy of Heisenberg \( \Delta x \sim 1/\Delta k_x \) allows.

6. ONE-DIMENSIONAL PHOTONIC CRYSTAL

We shall consider the propagation process of signal electromagnetic wave in the film in which the spatial polariton wave is exited by the powerful electromagnetic pump wave. As the result of Kerr nonlinearity the refractive index of dielectric medium in the areas around the maxima of the polariton wave,

\[
\psi_1 = \sqrt{\frac{\tilde{g}_f}{\tilde{g}_2}} \sin \left( \frac{b_1 (\sqrt{2} x, \tilde{k})}{\sqrt{2}} \right) \exp \left( -i \tilde{\Omega} t + i \ell \frac{\pi}{L} \right),
\]

that is excited by pump wave, will be higher than in the areas around the minima [6]. Thus, the one-dimensional superlattice forms by the potential \(-\sin^2 \left( \frac{b_1 (\sqrt{2} x, \tilde{k})}{\sqrt{2}} \right)\) in the thin dielectric film, i.e., the tapers with width \(-1\) \(-\tan^2 \left( \frac{\tilde{g}_2}{\sqrt{2}} \right)\) and the period \( \Lambda = 2K(\tilde{k}) \) appear in the film. The polariton signal wave \( \psi_1(t, \mathbf{r}) \exp \left( -i \omega t + i \ell (\pi / L) \right) \) scatters at the superlattice. The superlattice shape can be controlled by varying the external electrostatic field \( E_0 \).

The equation for the polariton signal wave has the form

\[
\frac{i}{\hbar} \frac{\partial \psi_1(t, \mathbf{r})}{\partial t} = - \frac{\hbar}{2m_{2f}} \nabla^2 \psi_1(t, \mathbf{r}) + \bar{\alpha} \psi_1(t, \mathbf{r}) \psi_1(t, \mathbf{r}) \psi_1(t, \mathbf{r}),
\]

(10)

where \( m_{2f} = \hbar \omega_2 / c^2 \). We shall rewrite Eq. (10) as

\[
\frac{d^2 \tilde{\psi}_2}{dx^2} + \left[ 2 \frac{\omega_2^2}{c^2} - \ell^2 \frac{\pi^2}{L^2} - \frac{\tilde{g}_f \omega_2 b_2^2}{\tilde{\Omega}} \sin^2 \left( \frac{b_1 (\sqrt{2} x, \tilde{k})}{\sqrt{2}} \right) \right] \tilde{\psi}_2 = 0.
\]

(11)

In Eq. (11) the functions \( U(X^*) = 2\omega_2^2 / c^2 - \ell^2 (\pi^2 / L^2) - (\tilde{g}_f \omega_2 b_2^2 / \tilde{\Omega}) \sin^2(X^*, \tilde{k}) \) (where \( X^* = b_1 (\sqrt{2} x, \tilde{k}) \)) have the real period \( \Lambda = 2K(\tilde{k}) \): \( U(X^* + \Lambda n) = U(X^*) \), \( n = 0, \pm 1, \pm 2, \ldots \). Equation (11) is the scattering-wave equation for the periodic potential. The exact solutions of Eq. (11) are the polynomials—Lame functions [11]. The physical model that is described by Eq. (11) is the one-dimensional photonic crystal with the variable width of superlattice. Equation (11) has the form

\[
\tilde{H}_0 \tilde{\psi}_2 + (\eta - V) \tilde{\psi}_2 = 0,
\]

(12)

where \( \tilde{H}_0 = d^2 / dx^2 \) is the unperturbed Hamiltonian, \( \eta = 2\omega_2^2 / c^2 - \ell^2 \pi^2 / L^2 \), and \( V(X^*) = \tilde{g}_f \omega_2 b_2^2 / \tilde{\Omega} \sin^2(X^*, \tilde{k}) \) is the perturbation.

Let us assume that periodic potential \( V \) is the weak perturbation [12]. At zero approximation if \( V = 0 \) the solutions of the unperturbed equation are the plane waves \( \psi_{20}(x) = \exp (\pm ik_{20} x) \). For the solution of Eq. (12) at zero approximation we shall take a linear combination of the plane waves \( \tilde{\psi}_{20}(x) = A \exp (ik_{20} x) + B \exp (-ik_{20} x) \) and we shall substitute it in Eq. (12):

\[
A(-k_{21}^2 + \eta - V) \exp (ik_{20} x) + B(-k_{21}^2 + \eta - V) \exp (-ik_{20} x) = 0.
\]

After that we multiply the equation by \( \exp (ik_{20} x) \), then by \( \exp (-ik_{20} x) \), and then integrate by \( X^* \) the received equations, taking into account the periodicity of potential. We obtain the combined equations for the stationary values \( A \) and \( B \):

\[
A(-k_{21}^2 + \eta - V_{11}) + BV_{12} = 0,
\]

\[
AV_{21} + B(-k_{21}^2 + \eta - V_{22}) = 0,
\]

(13)

where

\[
V_{11} = V_{22} = \omega_2 \frac{\tilde{g}_f b_2^2}{\tilde{\Omega}} \int_{-\Lambda/2}^{\Lambda/2} dX^* \sin^2(X^*, \tilde{k});
\]

\[
V_{12} = \omega_2 \frac{\tilde{g}_f b_2^2}{\tilde{\Omega}} \int_{-\Lambda/2}^{\Lambda/2} dX^* \exp \left( -i \frac{2 \sqrt{2} k_{21} x}{b_1 \sqrt{2} \tilde{k}} \right) \times \sin^2(X^*, \tilde{k}),
\]

\[
V_{21} = V_{12},
\]

are the matrix components of the perturbation operator in the first Brillouin zone, \( \Lambda/2 = K(\tilde{k}) \). From the requirement of nontrivial solution of the linear combined Eqs. (13):

\[
\psi_{20}(x) = \exp (\pm ik_{20} x).
\]
Fourier series

we find the parameter $\eta$ for the one-dimensional model at the first approximation:

$$\eta = k_{2x}^2 + V_{11} \pm \sqrt{|V_{12}(k_{2x})|^2},$$

where

$$\sqrt{|V_{12}(k_{2x})|^2} = \frac{\omega_0 b_0^2}{\Omega} \left[ \left( \int_{-\Delta/2}^{\Delta/2} dX' \sin^2(X', \tilde{k}) \right)^2 + \left( \int_{-\Delta/2}^{\Delta/2} dX' \cos^2(X', \tilde{k}) \right)^2 \right]^{1/2}.$$

If one presents the periodic potential in the form of the Fourier series $V(X') = \sum_{n=-\infty}^{\infty} V_n \exp(iK_nX')$, where

$$V_n = \frac{\tilde{g}_t \omega_0 b_0^2 \Delta / 2}{\tilde{\Omega}} \int_{-\Delta/2}^{\Delta/2} dX' \sin^2(X', \tilde{k}) \exp(-iK_nX')$$

are the amplitudes of harmonics, $K_n = 2\pi / \Lambda, n \neq 0$, it is possible to find the matrix components for the $n$th zone.

We can find the frequency $\omega_2 > 0$ from Eq. (15) and obtain the spectrum of the polaritons that excited by the signal wave in the Brillouin zone $k_{2x} \in [-\pi / \Lambda', \pi / \Lambda']$:

$$\omega_2 = \frac{\epsilon^2}{4} \left( V_{11}^2 \pm \sqrt{|V_{12}(k_{2x})|^2} \right)^2 + \left( \frac{V_{11} \pm \sqrt{|V_{12}(k_{2x})|^2}}{\omega_2} \right)^2 \right)^2,$$

where $V_{11} = V_{11}/\omega_2$, \sqrt{|V_{12}(k_{2x})|^2} = |V_{12}(k_{2x})|^2/\omega_2$, and $\Lambda' = 2\sqrt{2K_{2x}^2}/b_1 \tilde{g}_t$. In the Brillouin zone we can see two frequency branches (Fig. 2; $\omega_2 \times 10^{15} \text{s}^{-1}, k_{2x} \times 10^5 \text{cm}^{-1}$). The gap in the signal spectrum takes place at $\sqrt{2}/b_1 \tilde{g}_t = 1/2$. If the value of external electrostatic field $E_0$ increases, the coefficients $b_1$ and $\tilde{g}_t$ are increased (since $E_0$ enters $b_1$ and $\tilde{g}_t$ through the coefficients $\omega_0, a_1, a_2$ and, for example, we obtain the value $\sqrt{2}/b_1 \tilde{g}_t = 1$ (Fig. 3). There is the tendency to decrease in twice the spatial period of the spectrum along the Brillouin zone. The gap width $\Delta \omega = \omega_2 - \omega_2$ is determined by “contrast” between the refractive index of the superlattice tapers. Thus we can control the polariton profile spectrum $\omega_2(k_{2x})$ in the photonic crystal, varying the value of external electrostatic field $E_0$.

7. CONCLUSION

Thus the polariton quasi-condensate arises at the amorphous dielectric medium in the unclosed resonator. The quantum interacting spatial cnoidal waves are generated in the quasi-condensate. It is possible to create the quantum spatial waves with zeros of the field at the transverse axis of the resonator mirrors in the amorphous dielectric film with reflecting edges. The electromagnetic pump wave produces the superlattice with the controllable width by the external electrostatic field, i.e., the one-dimensional controllable photonic crystal is formed in the dielectric film. The one-dimensional photonic crystal with the controllable breadth of the allowed and forbidden spectral zones acts as perspective for the creation of the optical information transmission lines and for the optical processing in the whole.

REFERENCES